XVIII SemeAd
Seminários em Administração

novembro de 2015
ISSN 2177-3866

PORTFOLIO EVALUATION VOLATILITY TIMING AND REWARD TO RISK TIMING INVESTMENT STRATEGIES: THE BRAZILIAN CASE

ROBERT ALDO IQUIAPAZA COAGUILA
UFMG
riquiapaza@gmail.com

GUSTAVO FIUZA COSTA VAZ
UFMG
gustavo_f_vaz@hotmail.com

SERGIO LOURO BORGES
Universidade Federal de Juiz de Fora - Campus GV
SERGIO.BORGES@UFJF.EDU.BR

.

Área temática: Finanças
Tema: Técnicas de investimento

PORTFOLIO EVALUATION VOLATILITY TIMING AND REWARD TO RISK TIMING INVESTMENT STRATEGIES: THE BRAZILIAN CASE

Abstract

Volatility investment is growing as an alternative to traditional portfolio investment. This research aimed to verify the performance of the Volatility Timing (VT) and Reward to Risk Timing (RRT) strategies of portfolio selection on the Brazilian stock market. The assets employed in the analysis were those included in the Ibovespa Index in the period from January of 2004 through December of 2014. We used statistical and financial indicators to measure the performance of the strategies. It was possible to compare both strategies against the Minimum Variance (WVm) portfolio of Markowitz (1952), the Ibovespa (Ibov) and the Naïve portfolios. The Ibov and Naïve presented the lowest portfolios returns. In the other the hand the Wvm, VT4 and RRT4 had the highest results. The focus of this research was to find better portfolios that consistent preserves a low turnover, and the conclusion was that only two portfolios followed the requirements in the Brazilian scenario: the VT4 and the Wvm portfolios. At last, one of the most important results of this research is that the best choice of portfolio will depend upon the economic setting that the Brazilian market is presented, because some of the strategies had good results in specific periods.
Keywords: Portfolio selection, Volating Time strategy, Reward to Risk Timing strategy

1. Introduction

Several changes had occurred in the modern finance theory, having Harry Markowitz as the first mark of these changes with the article "Portfolio Selection" (1952). In that, the author realized that ordinary investors usually followed the rule of choosing the higher return and lower volatility assets, which could result in a risky allocation strategy (MARKOWITZ, 1952).

Assuming that risk is the quantification of the chance of something happening differently from the expected, and return is the expected payoff on the risk of a chosen asset, Markowitz (1952) suggested a new rule of investment called diversification. In this case, the investor chooses several assets, creating a portfolio that tries to reduce the risk among the chosen assets. Those portfolios created following the Markowitz's diversification theory are called Mean-Variance (MV).

Even though the MV - focused on identifying the efficient frontier – had been used in many recent studies, many criticisms were made to this theory since its proposition. Tu and Zhou (2011), for example, claim that the results obtained by MV technique may contain estimation errors given the use of historical data in its calculations. To correct these errors, it has been created over time different methodologies using forms of optimization that can achieve minimum risk with higher returns than those from the MV approach as we can see in DeMigel et al.(2009) and Fletcher (2011).

On the other hand, Maillard, Roncalli and Teiletche (2008) believe that the market still has a large fraction of investors that prefer heuristic methods. Also the problem of estimation errors when forecasting returns by the unrestricted Mean-Variance approach, as pointed by Sharma (2015) or Madeiros, Passos and Vasconcelos(2014) led investors to use one exclusive case of the Mean-Variance method, the Minimum-Variance (Wvm) portfolio, thus focusing on a volatility (risk) based strategy.

In contrast to those optimized portfolios, the Naïve – innocent portfolio-, distributes the investment equally among the N selected assets (TU, ZHOU, 2011). Several studies, such as Tu and Zhou (2011), DeMigel et al. (2009) and Fletcher (2011) claim that, despite not having a theory behind, the naïve portfolio performs well when compared to other portfolios, often being more interesting and desired.

To find high levels of the performance indicators, achieving very favorable returns and risks, authors increasingly used robust forms of portfolio optimization. For example, methods of *"shrinkage"* or tuning-parameters as in Ledoit and Wolf (2003), Tu and Zhou (2011), and Kirby and Ostdiek (2012b) trying to reduce estimation errors. Despite finding some interesting results, some techniques also increase asset turnover. Therefore, the high turnover of those methods also brings high costs for their portfolios, thus made their strategies not desirable as expected.

Seeking to reduce such problems, Kirby and Ostdiek (2012a) created two different methods that use a combination of the Naïve and the Minimum-Variance portfolio, in which the weights of the assets are limited to never be negative: Volatility Timing (VT) and Reward to Risk (RRT). These two methods were tested in foreign markets, obtaining very good results for the investors. Because they do not need optimization, the calculations of the portfolio weights are simpler and faster compared to other approaches. Furthermore, at the end of the Kirby and Ostdiek (2012a) experiments, the performance indicators found as their results for both methodologies exceeded the innocent portfolio and also had low levels of turnover hence low transaction costs.

As long as the stock markets have peculiarities, it is important to verify if the new methods can be applied bringing the same results in other countries. Based on that, this research aims to verify the performance of the VT and RRT models proposed by Kirby and

Ostdiek (2012a) compared with the Naïve and Wvm ones applied to the Brazilian stock market.

The next section will present the theoretical framework; after that, the analytical framework will be presented; the results and discussions are shown on topic 4; at the last one we will present conclusion of this research, its limitations and some suggestions for future ones.

2. Theoretical Framework

Markowitz deepened his studies in finance and capital markets, publishing a world-renowned article titled "Portfolio Selection" in 1952. The diversification rule created by Markowitz (1952) in that article aimed to eliminate the strategies that separated the expected return of an asset's risk. For the author, risk and return are extremely correlated traits in which if the asset has a high risk it is required by the investor a higher return on it.

> "The hypothesis (or maximum) that the investor would (or should) maximize the return must be rejected. If we ignore the imperfections of the market, this rule would mean that there is never a diversified portfolio that is preferred over those who are not diversified. Diversification is observed and sensitive; the rule of behavior that does not imply the superiority of diversification should be rejected, so how much chance as maximum." (Markowitz, 1952, p77)

The possibility of risk reduction with the diversification, the risk calculation (or estimation) turned into a very important issue in finance. Risk can be approximated by various ways, but variance and standard deviation frequently are used by many authors in their calculations. Markowitz as well realized in his studies the possible use of correlation and covariance to represent the relationship between the movements of the assets. Separate securities in the market yet have a relationship between their movements which directly affect their volatilities, which could be calculated and added to the formulations to achieve the portfolio risk. Given by:

$$\sigma_{ij} = E\{(R_i - \bar{R}_i) * (R_j - \bar{R}_j)\} \text{ or } \sigma_{ij} = \rho_{ij} * \sigma_i * \sigma_j \quad (1)$$

$$\text{and: } \rho_{ij} = \sigma_{ij} \div (\sigma_i * \sigma_j) \quad \text{where: } -1 \leq \rho_{ij} \leq 1 \quad (2)$$

Where σ_{ij} represent the covariance of an asset i with another asset j; R_i the expected return on asset i; $\bar{R}_i$ the average return of asset i; $(R_i - \bar{R}_i)$ the deviation of returns; ρ_{ij} the correlation between assets i and j; $\sigma_i$and σ_j are the standard deviations of assets i and j respectively.

In addition, Markowitz (1952) proposed that to diversify a portfolio, investor should not only add new assets, but should also make sure that the correlations between its assets were closest to -1 as possible, which means that two assets have perfect inverse correlation. In this case, if an asset reaches 10% return the other will have -10%, but if the correlation is equal to 1, if one asset had 10% return the other would have the same return. Santos e Tessari (2012) enforced the importance of this approach correlation and covariance rectifying the relationship between risks and returns, eliminating the idea of independence between the risks of the assets within a portfolio.

From these findings, Markowitz (1952) calculated the optimal weights for a portfolio that he would call efficient. To show this, he simplified his theory using only portfolios of three and four assets, and a stationary price process to calculate the historical returns (SHARMA, 2015) to create a covariance matrix. The mathematical formula used to calculate the variance of a portfolio is:

$$\sigma_p^2 = X' * M * X \quad (3)$$

Where X is the weight vector and X' it's the transposed matrix, M is the matrix created from the covariance between each asset in the portfolio.

To calculate the returns and weights for the efficient portfolio, historical returns and their arithmetic mean within a given timeline were used. The average returns vector is then multiplied by the inverse of the covariance matrix to find the weight vector.

$$X = M^{-1} * \bar{r} \quad (4)$$

Where X is the weight vector; M^{-1} is the inverse covariance matrix, and $\bar{r}$ the vector of average returns.

The efficient frontier of Markowitz (1952) is the connection of all efficient portfolios calculated from the proposed methods from the minimum variance portfolio, which is found from a return vector of "ones" to the Tangency Portfolio (TP) that has excess returns as the return vector given by $\overline{R}_i - R_f$, where $\overline{R}_i$ is the average return of asset i and R_f is the return of a risk-free asset. This combination of the two portfolios is given by a k weighting in risky assets from the TP and (*1 -k*) weighting in the Minimum-variance (Wvm).

The efficient frontier is found from two basic concepts in the literature and should be followed: (1) the investor will always prefer the portfolio with the highest possible return for a given level of risk, and (2) the investor always prefer the portfolio with minimum risk for a given level of return. (Markowitz, 1952)

The theories of Markowitz (1952) formed the foundation for the evolution of modern finance and so arose several criticisms of his methods. Over time some researches found empirical evidences that the author's investment strategy was less efficient than other methods, as presented by Tu and Zhou (2011), Kirby and Ostdiek (2012) among others.

Maillard et al.(2008) reinforce the idea that the MV is a very attractive strategy, but concentrate its portfolios in a few subsets of securities. Also this approach is over sensitive to input changes and therefore the problems that Markowitz (1952) encountered inspired many authors to use different approaches such as risk-based methods (Sharma, 2015).

According to Lee (2011) the financial crisis of 2008 made investors question the theories in the construction of their portfolio. To diverge from the forecasting of returns, that is known to have great estimation errors some portfolios studies focused solely on the forecasting of the risk as an input (LEDOIT, WOLF, 2003).

Along the time, different kinds of strategies where proposed. One strategy that does not use means as an input is the Naïve portfolio. Duchin and Levy (2009) described it as a strategy that has been used by Babylonian Talmud. Tu and Zhou (2011) also claim that this strategy has around 1500 years old. It is determined by equal division of an amount to be invested among the assets chosen by the investor: $\frac{1}{N}$. Despite using a non complex mathematical approach the results are surprising when compared to other portfolios such as the unconstrained MV. The Naïve portfolio is often superior to other portfolios and when it does not, optimization methods of mean-variance portfolios were required to obtain better performance. Fletcher (2011) says that the shortcomings from practical implementation of the mean-variance analysis are the estimation of risks posed by the forecasts of the covariance matrix and returns on assets.

The theory of Markowitz (1952) helped in the understanding of several existing factors that should be analyzed prior to any investment in the capital market, but this did not solve the problems of the unpredictability of the market and consequently the estimation errors that come from the estimates of the matrix covariance and the expected returns. William Sharpe (1964) sought to eliminate as much of the returns estimation problems creating from the models of Markowitz (1952), a new model called the CAPM (Capital Asset Pricing Model).

This new model had only one factor, but was considered an evolution in estimation techniques of return of assets. Given by:

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f) \quad (5)$$

$$\beta_i = \frac{Cov(i,m)}{Var(m)} \quad (6)$$

Where E (R_i) represent the expected return for the chosen asset; R_f the return on risk-free asset; β_i is the asset sensitivity to the market; E (R_m) the return expected by the market; Cov (i, m) the covariance between the return of the asset and the market one and Var (m) the variance of the market return.

The beta, at that moment, became a more accurate measure of risk and soon came to be used more frequently than the standard deviation proposed by Markowitz. Despite being a great model, the CAPM still had difficulties to express some information about other important characteristics of assets. Fama and French (1993) showed in their studies that in most cases factors such as size of the companies or the book-to-market index had better evidences that explained the sensitivity of the average returns of the securities. Carhart (1997) added the *momentum* factor to the CAPM used by Fama and French (1993) and obtained even greater results in the average return estimation. The four-factor model Fama and French and Carhart (FFC) can be determined by:

$$R_i = R_f + \beta_i(R_m - R_f) + \beta_{SMB}(SMB) + \beta_{HML}(HML) + \beta_{WML}(WML) \quad (7)$$

On what: R_i is the expected return on the asset i; R_f the return of risk-free asset; R_m the return of the market portfolio; β_i is the beta sensitivity of the market portfolio and the portfolio i; β_{SMB} the beta that represents the size factor; β_{HML} is the beta for book-to-market factor; β_{WML} the beta momentum factor; SMB the premium for the size factor; HML is the premium for the fraction $\frac{VC}{VM}$ (book value over market value); and WML is the momentum effect.

Caldeira, Moura and Santos (2013) emphasized that "Models of factors emerge as a promising alternative to solve the problem of dimensionality and offload the econometric estimation process". The CAPM with FFC factors paved the way to facilitate the process of optimization of portfolios, finding ways to achieve higher returns and lower risks. For many years were tested various types of optimized portfolios in different countries, all being compared to the naïve portfolio. However, despite being superior in most tests, one of its characteristics prevented the equally weighted portfolio to be set aside, this feature is called turnover.

The turnover are all transactions of financial assets made by an investor, from purchase to selling them, generating costs on all drives. In any optimization process is demanded a greater number of transactions to be able to maintain a high level of return and low risk. Thus transaction costs become problematic and may cause the chosen method to be undesirable.

In effort to reduce the turnover problem Kirby and Ostdiek (2012) propose two alternatives methods: the Volatility Timing (VT) and Reward to Risk Timing (RRT). The VT is a method in which it's observed and corrected two characteristics - the risk that is brought from the estimation error of the expected returns and the high transaction costs from the portfolio optimization methods. Within this methodology "portfolios are rebalanced monthly based only on changes in the estimated conditional volatilities of asset returns" (KIRBY, OSTDIEK, 2012,pp. 439). As seen in Maillard et al. (2008) the combination of the contributions of the Naïve and Minimum-Variance portfolios, since this last one works with a vector of "ones" instead of expected returns, can help lower the estimation risk and follow the path of the risk-based portfolios that Lee (2011) presented in his article.

This VT strategy uses only temporal variances of assets to calculate their weights within the portfolio. Despite having a fairly simple formula, compared to other methods, the Volatility Timing overcame the innocent diversification, and managed to remain superior in transaction costs in the dataset tested by Kirby and Ostdiek (2012). VT ignores information about expected returns, but gains in reduction of risk estimation.

However, the Reward to Risk Timing (RRT) maintains the mitigation of risk estimation, using only the variances of asset returns, but also incorporates significant information on returns in some of its methods. Using the four-factor model of Fama, French (1991, 1992) and Carhart (1997), RRT were proposed in two different analyses: one includes the mean of the asset returns and the other $\bar{\beta}$ (average beta calculated by the number of factors used) thus increasing information that enhances the methodology and does not bring the high estimation errors that comes from average returns. These two strategies, especially the RRT, succeeded in several studies in overcoming the $\frac{1}{N}$ method and also in controlling the turnover of their portfolios. It is important emphasize that these methods were not tested on the Brazilian market data before this study. Such models will be analyzed in more detail in the next section.

3. Analytical Reference

For the analysis proposed into this research, the Brazilian stock market was represented by the group of assets that compose the Bovespa Index (Ibovespa). It was chosen based on the propused by Iquiapaza at el. (2014) updated until December of 2014[i], for being a representative of the national stock market (BM&FBOVESPA, 2015). This theoretical portfolio is rebalanced along the year, so the assets inside the portfolio change after each rebalancing. The number of the total assets that went through the portfolio was 113, and it was chosen only the securities that presented at least twenty full months continuously cotted with their price rates from January of 2004 to December of 2014. In the end of selections, these assets were used in the construction of the Volatility Timing and Reward to Risk Timing strategies.

Kirby and Ostdiek (2012a) refer to four essential characteristics of both VT and RRT: (i) they do not use the inversion of the covariance matrix; (ii) they do not generate negative weights[ii]; (iii) they do not use optimization; and (iv) they adjust the sensitivity of the weights to the volatility changes with a parameter η.

The VT strategy can be considerate as obtained by very simple calculations because it uses only two basic indicators in its formula: the variance and the parameter that determines the aggressiveness of the investor – representing how faster the investor will review their portfolios rebalancing it.

The results of VT method are obtained by the following formula:

$$\omega_{it} = \frac{(\frac{1}{\sigma_{it}^2})^\eta}{\sum_{i=1}^{N}(\frac{1}{\sigma_{it}^2})^\eta} \quad (8)$$

Where ω_i is the weight of asset i; σ_i^2 the variance of asset i; η the measures of the aggressiveness in which the investor balances the assets of its portfolio; t is the period in which the weights are being calculated. According to Kirby and Ostdiek (2012a) the parameter is a proxy of the weights in relation to changes in the volatility of the assets in time. If $\eta = 0$ the result is the innocent portfolio 1 / N, but if we put $\eta \rightarrow \infty$ the weight of the asset less variance will tend to 1. Thus we follow the idea of $\eta > 1$, which will compensate for the loss of information by annulling the correlations between asset returns. For this study it was considerate η assuming the values {1,2,4} for the comparison purpose.

The other strategy tested is the RRT, that comparatively is slightly more robust than the VT because try to not ignore the information about the expected returns. Kirby and Ostdiek (2012a) discuss the evolution of this strategy. On the assumption that the estimated pair-wise correlations between the excess risky-asset return are 0, the variance and the returns were used in the formula above to calculate the RRT using the average excess of returns of the assets (RRT).

$$\omega_{it} = \frac{(\hat{\mu}_{it}/\hat{\sigma}_{it}^2)^\eta}{\sum_{i=1}^N(\hat{\mu}_{it}/\hat{\sigma}_{it}^2)^\eta} \quad (9)$$

Where ω_{it} is the weight of the assets in the portfolio, $\hat{\mu}_{it}$ the mean of the excess of returns and $\hat{\sigma}_{it}^2$ is the variance from the asset i along the time t.

Kirby and Ostdiek (2012a) discussed that because the expected return are typically estimated with less precision than variances, the strategy is likely to entail significantly higher levels of estimation risk than the VT strategies, because of the extreme weights acquired if the means were to be negative for some assets, possibly causing the denominator to get close to zero.

To avoid this distortion, they proposed the RRT with the positive constraint (RRTK), where $\mu_{it} \geq 0$, which could be calculated by

$$\omega_{it} = \frac{(\hat{\mu}_{it}^+/\hat{\sigma}_{it}^2)^\eta}{\sum_{i=1}^N(\hat{\mu}_{it}^+/\hat{\sigma}_{it}^2)^\eta} \quad (10)$$

Where the $\hat{\mu}_{it}^+$ is the mean of the asset *i* conditioned to the positivity. Using only the positive average excess returns assume that the investor will not work with rates that are lower than the risk free asset.

It was also proposed an even more robust RRT strategy based on the four factor model of the CAPM suggested by Fama and French (1992, 1993) and Carhart (1997). The RRT beta uses the average beta of the four factors found in a regression to substitute the mean of excess returns on the first type of RRT. The investors assume that the factors are constant so they calculate a simple mean of the four betas, but also that they will only use positive betas on their calculations. In this article the betas were tested by significance in p-value of 10% and then added two different types beta RRTs called RRTbm and RRTbms, where the *bm* is all the positive mean betas, and the *bms* are all the significant mean betas found in the regression of the CAPM 4-Factor.

$$\omega_{it} = \frac{(\bar{\beta}_{it}^+/\hat{\sigma}_{it}^2)^\eta}{\sum_{i=1}^N(\bar{\beta}_{it}^+/\hat{\sigma}_{it}^2)^\eta} \quad (11)$$

The $\bar{\beta}_{it}^+$ is the mean positive beta of the asset *i* conditioned to the positivity restriction in the period t.

All the strategies will be compared to the Naïve portfolio, Ibovespa, CDI and Minimum-Variance. For the tests purpose, we also added a different type of Naïve portfolio that constraints the investor to only choose securities that have positive excess returns and we called it Wnp. As performance indicators for all the strategies we used the accumulated return on portfolio, average return on the portfolio, standard deviation, Sharpe Ratio, Herfindahl Index, turnover and the Breakeven as used by Santos and Tessari (2012) and Iquiapaza et al. (2014).

Furthermore, to check if there is different tendency along the whole period, the strategies will be analyzed according to three specific time periods: March 2009 through March 2011, April 2011 through December 2012, and January 2013 through December 2014.

The entire compilation of the databases and formulas for each strategy were modeled in the statistical software R, from R Project for Statistical Computing. The formulas were created throughout the project taking into account the programming created by Iquiapaza et al.(2014), using 132 months and a rolling window equals 60, resulting 72 windows with different assets weights that composed the Ibovespa index portfolio. CDI rates were used as a proxy for the risk free asset used on all calculations throughout the project. The returns and the asset prices were obtained from the Economatica database and BM&FBovespa from January of 2004 to December of 2014.

For a better view, the table 1 summarizes all the basics information for the strategies used along this research.

Table 1 – Descriptive information about the strategies analyzed.

Strategies	Symbols	Description
Naïve Portfolio	Wn and Wnp	Portfolio created with equal division between the assets. The Wnp has a restriction of allocation to only positive excess returns assets.
Minimum-Variance Portfolio	Wvm	Minimum-Variance portfolio built by Markowitz (1952) using rolling window forecasting.
Volatility Timing	VT1, VT2 and VT4	Strategy that uses only variance forecasting to calculate portfolio weights. The number in front of the symbol "VT" represents the value of the tuning parameter η used for the strategy.
Reward to Risk Timing with and without Positive excess returns restriction	RRT1, RRT2, RRT4, RTk1, RRTk2, RRTk4	Strategy that adds more information to the VT, such as the mean excess returns for the RRT, and with the positive constraint for the excess of returns for RRTk. The number in front of each symbol also represents the tuning parameter η used for the strategy.
Reward to Risk Timing with Average Beta and Significative Average Betas	RRTbm2, RRTbm4, RRTbms2, RRTbms4	These adds to the VT strategy information, the average positive Beta calculated from a regression of the CAPM 4-Factor model in the case of the RRTbm, and restriction of only significant mean Betas for RRTbms. The number in front of each symbol also represents the tuning parameter η used for the strategy.
Ibovespa Index	IBov	A theoretical portfolio created for indexation of the BM&Fbovespa stock market in Brazil.
Risk Free Asset	CDI	This is a proxy for the risk free asset used in the article.

The next section will present the data characteristics and the results of the strategies applied to the Brazilian dataset.

4. Results and discussions

The Bovespa Index (Ibovespa) is a theoretical portfolio that indicates the average performance of the most negotiable assets of the BM&FBovespa. It also uses the assets that most represents the Brazilian market at a given time, therefore the assets that compose the Ibovespa portfolio are rebalanced periodically, changing its composition every trimester. Only the BM&FBovespa's securities can be part of this index and they have to follow a set of rules to be qualified as part of the portfolio (BM&FBOVESPA, 2014).

Along the period analyzed, some assets were included on IBovespa more frequently, considering the minimum permanency of twenty months in a row- that is one of the restrictions imposed on the dataset used. It is important to say that since the rebalancing of the Ibovespa is quarterly some of the selections had the same set of assets, but after each rebalancing those assets could change.

The high frequency of some assets could be diagnosed by at least one of two characteristics: (i) a high participation in the Brazilian market such as size of the company or the importance of that company to the Brazilian economy, or (ii) a great performance by the company during the process of composition of the index. Some examples are Petrobras (PETR3, PETR4) that holds an oil monopoly in Brazil which gives it a high degree of representativeness in its national market even though in the past year the company has been through one of its worse financial periods. Itaú Unibanco (ITSA4) in the other hand has had a great performance, which could indicate its high frequency inside the Ibovespa index.

In the case of the less frequent assets, we can say that they are less representative, did not perform well during the rebalancing stages of the index or maybe they passed the criteria to be part of the portfolio because of an extraordinary event. From these, the assets from Companhia de Bebidas das Americas – AMBEV(ABEV3) one of the biggest beer enterprise in the world, Redecard (RDCD3) and UNIBANCO (UBBR11) were the three which had the lowest frequency on the Index (Ibovespa) .

Presented the dataset main features we can start the presentation of the results obtained by the strategies tests. But before starting the performance analysis, Chart 1 shows the performance of the strategies along the period of the 72 rolling windows. From that is visible that the portfolios followed a very close tendency of the market until March of 2011 at a point that they started to differentiate. It is also visible that the Wvm, VT4 and RRT4 strategies had the highest results since September of 2012.

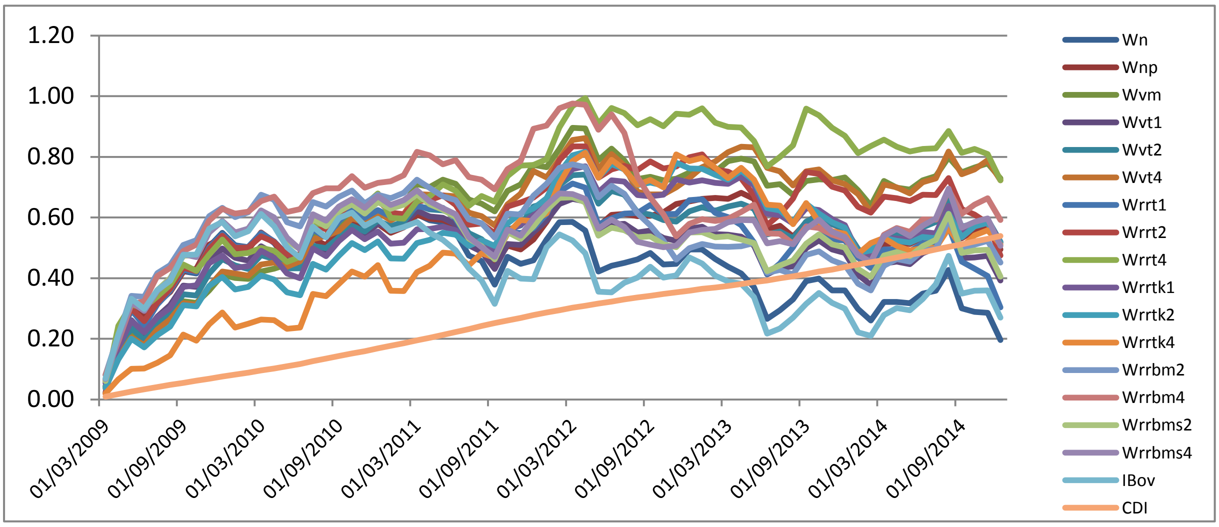

Chart 1 – The strategies returns along the portfolios.
Search: Research results

After presented overview of the results, let's take a deeper look at the strategies. At first they were formed the portfolios, then returns of the VT and RRT strategies and its variables were used over the rolling windows with their set of securities. The Table 2 present the descriptive data from the portfolios analyzed.

As presented in Table 2, all the strategies had a negative return as a minimum, where WRRTbm4 had the worse one with -14% in one of the 72 portfolios, followed by the Naïve, WRRTbms2 and 4 with -13.48%, -12.85% and -12.91%, respectively.

The highest maximum return was presented by the unconstraint RRT4 with 20.34%, followed by the Naïve, RRT1 and RRT2 with 16.13%, 17.33% and 18.94%, in this order. Analyzing the mean, the lowest results were the Naïve and Ibov reaching 0.28% and 0.39% in contrast the highest were the Minimum-variance (Wvm), VT4 and RRT4 with 1.04% and the other two with 1,03%, respectively.

Table 2 – Descriptive results from the portfolios- Minimum, Median, Mean, Max and 1st quarter and 3rd quarter .

Indicators	Wn	Wnp	Wvm	Wvt1	Wvt2	Wvt4
Min.	-0,13480	-0,10670	-0,10460	-0,11180	-0,09770	-0,10100
1st Qu.	-0,03020	-0,01880	-0,01190	-0,02360	-0,01740	-0,01280
Median	0,00016	0,00610	0,00930	0,00190	0,00750	0,01150
Mean	0,00279	0,00710	0,01040	0,00560	0,00740	0,01030
3rd Qu.	0,03788	0,03410	0,04030	0,03990	0,03790	0,03720
Max.	0,16130	0,14710	0,11290	0,13460	0,11790	0,10100
Indicators	**Wrrt1**	**Wrrt2**	**Wrrt4**	**Wrrtk1**	**Wrrtk2**	**Wrrtk4**
Min.	-0,12200	-0,10650	-0,08600	-0,09458	-0,09002	-0,08492
1st Qu.	-0,02660	-0,02170	-0,02130	-0,02169	-0,01963	-0,02508
Median	-0,00090	0,00200	0,01053	0,00399	0,00858	0,00913
Mean	0,00430	0,00700	0,01033	0,00730	0,00739	0,00729
3rd Qu.	0,04210	0,04140	0,03917	0,03922	0,03870	0,03894
Max.	0,17330	0,18940	0,20337	0,11749	0,10389	0,11005
Indicators	**Wrrbm2**	**Wrrbm4**	**Wrrbms2**	**Wrrbms4**	**IBov**	**CDI**
Min.	-0,09956	-0,14498	-0,12860	-0,12911	-0,12620	0,00480
1st Qu.	-0,02516	-0,01978	-0,02478	-0,02399	-0,03610	0,00690
Median	0,00709	0,01206	0,00250	0,00609	0,00110	0,00780
Mean	0,00645	0,00845	0,00578	0,00723	0,00390	0,00770
3rd Qu.	0,03751	0,03954	0,03864	0,03792	0,03680	0,00860
Max.	0,13047	0,12406	0,14309	0,13270	0,14450	0,01100

Search: Research results

To evaluate the performance of the strategies and compare them to each other some indicators were used: accumulated return on portfolio, average return on the portfolio, standard deviation, Sharpe Ratio, Herfindahl Index, turnover - as used by Santos and Tessari (2012) and Iquiapaza et al. (2014) - and the Breakeven. The results of performance are presented on Table 3.

Following the bad results seen before, the Ibov and Naïve presented the lowest portfolios returns with 10.15% and 16.67%. In the other hand the Wvm, VT4 and RRT4 in this order, had the highest results presenting returns of, 94.97%, 92, 13%, 87, 84%.

A very interesting result is the very close standard deviation along all the strategies, fluctuating from 4.1% until 5.35%, that is not very far from the 5.79% presented by the Ibovespa.

Related to the indexes calculated, only 4 strategies presented positive Sharpe Indexes of, 6.6%, 5.98%, 5.08% and 1.42% for Wvm, VT4, RRT4 and RRTbm4 respectively, opposing the negative results from the other portfolios. Even though the results were positive they were still low for any good consideration.

Table 3– Monthly Financial indicator for the portfolios created.

Indicators	Wn	Wnp	Wvm	Wvt1	Wvt2	Wvt4
Accumulated return	0,10150	0,50650	0,94970	0,36890	0,56810	0,92130
Mean	0,00280	0,00710	0,01040	0,00560	0,00740	0,01030
Sd	0,05350	0,04940	0,04100	0,04750	0,04470	0,04380
Betas	0,88730	0,77960	0,49580	0,76510	0,66050	0,51670
IS	-0,09160	-0,01280	0,06610	-0,04400	-0,00600	0,05980
IR	-0,06480	0,13520	0,15860	0,08020	0,11790	0,15200
IHerfindahl	0,00000	0,02660	0,14600	0,00700	0,03440	0,17170
Turnover	0,01800	0,11560	0,09070	0,02590	0,03940	0,06550
Breakeven	0,10940	0,03090	0,05860	0,11890	0,09760	0,08120
Indicators	**Wrrt1**	**Wrrt2**	**Wrrt4**	**Wrrtk1**	**Wrrtk2**	**Wrrtk4**
Accumulated return	0,22860	0,46290	0,87840	0,54940	0,56400	0,54830
Mean	0,00430	0,00680	0,01030	0,00730	0,00740	0,00730
Sd	0,05330	0,05220	0,05170	0,04580	0,04460	0,04570
Betas	0,85020	0,79270	0,66820	0,67330	0,59740	0,46600
IS	-0,06310	-0,01770	0,05080	-0,00860	-0,00700	-0,00900
IR	0,02190	0,10560	0,16450	0,11290	0,09660	0,07130
IHerfindahl	0,01270	0,05340	0,23480	0,06340	0,14170	0,35320
Turnover	0,10460	0,14300	0,18090	0,10800	0,12520	0,14480
Breakeven	0,02770	0,03150	0,03600	0,03150	0,03060	0,02990
Indicators	**Wrrbm2**	**Wrrbm4**	**Wrrbms2**	**Wrrbms4**	**IBov**	
Accumulated return	0,43390	0,63800	0,36610	0,50760	0,16670	
Mean	0,00640	0,00840	0,00580	0,00720	0,00390	
Sd	0,05130	0,05270	0,05170	0,05240	0,05790	
Betas	0,76600	0,60670	0,85400	0,82790	1,00000	
IS	-0,02450	0,01420	-0,03710	-0,00900	-0,06640	
IR	0,08930	0,10120	0,11100	0,14410	0,00000	
IHerfindahl	0,04400	0,15840	0,01830	0,07550	NA	
Turnover	0,07470	0,11280	0,07380	0,11590	NA	
Breakeven	0,04050	0,03080	0,04140	0,03060	NA	

Search: Research results

Tables 4, 5 and 6 were designed to show the annualized returns, standard deviation, Sharpe Ratio and Worst Drawdown according to a different time period than the whole set of 132 months. Looking for a better explanation of the relations along the whole period analyzed, we separated it in three parts: March 2009 through March 2011, then April 2011 through December 2012, and finally January 2013 through December 2014.

Table-4 – Financial indicators for the period from March 2009 through March 2011

Indicators	Wn	Wnp	Wvm	Wvt1	Wvt2	Wvt4
Annualized Return	0,31580	0,31490	0,38970	0,32810	0,34280	0,36980
Annualized Sd	0,18300	0,17970	0,12170	0,15750	0,13940	0,11720
Annualized Sharpe (Rf=9.24%)	1,09670	1,11160	2,20780	1,34610	1,61810	2,13580
Worst Drawdown	0,09460	0,10560	0,04290	0,07070	0,04330	0,02580
Calmar Ratio	3,33970	2,98080	9,08910	4,64210	7,91700	14,33310
Indicators	**Wrrt1**	**Wrrt2**	**Wrrt4**	**Wrrtk1**	**Wrrtk2**	**Wrrtk4**
Annualized Return	0,33120	0,34160	0,33970	0,28910	0,26420	0,21010
Annualized Sd	0,18680	0,18940	0,19440	0,16500	0,15040	0,14030
Annualized Sharpe (Rf=9.24%)	1,14950	1,18420	1,14510	1,06730	1,01860	0,73750
Worst Drawdown	0,07680	0,06330	0,08360	0,08800	0,06470	0,08620
Calmar Ratio	4,31230	5,39500	4,06140	3,28680	4,08300	2,43580
Indicators	**Wrrbm2**	**Wrrbm4**	**Wrrbms2**	**Wrrbms4**	**IBov**	**CDI**
Annualized Return	0,38960	0,45650	0,35630	0,36420	0,29630	0,09740
Annualized Sd	0,17490	0,14750	0,18480	0,18950	0,20100	0,00330
Annualized Sharpe (Rf=9.24%)	1,53510	2,23720	1,28710	1,29300	0,90860	0,00000
Worst Drawdown	0,10170	0,05120	0,11970	0,12330	0,13750	0,00000
Calmar Ratio	3,82930	8,91780	2,97610	2,95380	2,15490	∞

Search: Research results

During the first period (Table 4) the strategies had a growing tendency, but they were all close together, the second period (Table 5) the difference between their results started to increase and follow a falling tendency. Going into the third period (Table 6) most of the portfolios went into a convergence leaving 4 other portfolios to be considered the best.

All these tendencies could have been explained by economic characteristics of the Brazilian market of that specific time period such as the economic boom during 2009 and 2010, and the economic collapse on the later periods. These explanations aren't the scope of this article so it will leave an opportunity for later studies.

After checking the results it was possible to conclude that the best strategies in an overall manner were Wvm, VT4 and RRT4 in this order with returns of, 94.97%, 92,13%, 87,84%. They also had a relatively low risk. These three strategies also had the best Sharpe ratios, but the main focus of this project was the turnover rates. Looking at that perspective the VT strategies went accordingly to what was expected, a turnover that followed closely the Naïve portfolio. The 1/N had a turnover of 1.8% and the VT (1), (2) and (4) had ratios of 2.6%. 3.9% and 6.5%, respectively, compared to a 9.07% of the Wvm portfolio.

RRT is being analyzed aside because it did not follow the expected low turnover. The only two RRTs that had turnover under 10% were bm2 and bms2 with 7.5% and 7.4%. They also had worse results than the VT, counting negative Sharpe Ratios, putting aside only the RRT4 and RRTbm4 in that requisite. But even then only the RRT4 had a high return with 87.84%. Although that last strategy had better results they did not solve the main objective of this article which is the maintenance of a low turnover while achieving higher returns and lower risks.

Table-5 – Financial indicators for the period from April 2011 through December 2012

Indicators	Wn	Wnp	Wvm	Wvt1	Wvt2	Wvt4
Annualized Return	-0,07900	0,01680	0,01030	-0,04400	-0,02200	0,01680
Annualized Sd	0,18040	0,13790	0,15560	0,15140	0,14910	0,16760
Annualized Sharpe (Rf=9.24%)	-0,88800	-0,52100	-0,50100	-0,84300	-0,72200	-0,43000
Worst Drawdown	0,21320	0,15530	0,16670	0,17870	0,15050	0,17910
Calmar Ratio	-0,37200	0,10810	0,06190	-0,24500	-0,14600	0,09370
Indicators	**Wrrt1**	**Wrrt2**	**Wrrt4**	**Wrrtk1**	**Wrrtk2**	**Wrrtk4**
Annualized Return	-0,00100	0,07400	0,16240	0,08230	0,13810	0,21330
Annualized Sd	0,16530	0,15370	0,15920	0,13470	0,14880	0,17470
Annualized Sharpe (Rf=9.24%)	-0,53500	-0,12500	0,38950	-0,08680	0,26650	0,62330
Worst Drawdown	0,16370	0,12100	0,09470	0,09870	0,11080	0,11440
Calmar Ratio	-0,00800	0,61190	1,71430	0,83380	1,24650	1,86530
Indicators	**Wrrbm2**	**Wrrbm4**	**Wrrbms2**	**Wrrbms4**	**IBov**	**CDI**
Annualized Return	-0,13350	-0,14610	-0,08010	-0,08770	-0,08300	0,09800
Annualized Sd	0,16620	0,20760	0,15560	0,15500	0,19660	0,00550
Annualized Sharpe (Rf=9.24%)	-1,26480	-1,06880	-1,03480	-1,08420	-0,83000	0,00000
Worst Drawdown	0,27920	0,37080	0,19710	0,20170	0,24270	0,00000
Calmar Ratio	-0,47840	-0,39390	-0,40640	-0,43480	-0,34000	∞

Search: Research results

Table-6 - Financial indicators for the period from January 2013 through December 2014

Indicators	Wn	Wnp	Wvm	Wvt1	Wvt2	Wvt4
Annualized Return	-0,15200	-0,09100	-0,01800	-0,09500	-0,06100	-0,01600
Annualized Sd	0,17350	0,17640	0,13200	0,16660	0,15970	0,15690
Annualized Sharpe (Rf=9.24%)	-1,31900	-0,97400	-0,79600	-1,05500	-0,90500	-0,65800
Worst Drawdown	0,28160	0,22350	0,15040	0,18530	0,17870	0,19960
Calmar Ratio	-0,54100	-0,40500	-0,11800	-0,51100	-0,34000	-0,07900
Indicators	**Wrrt1**	**Wrrt2**	**Wrrt4**	**Wrrtk1**	**Wrrtk2**	**Wrrtk4**
Annualized Return	-0,17600	-0,16300	-0,11400	-0,10840	-0,12520	-0,13860
Annualized Sd	0,17740	0,17370	0,16390	0,15860	0,14990	0,14620
Annualized Sharpe (Rf=9.24%)	-1,41300	-1,37500	-1,18100	-1,18810	-1,35960	-1,47760
Worst Drawdown	0,32360	0,30600	0,23190	0,23640	0,25250	0,28250
Calmar Ratio	-0,54500	-0,53400	-0,49200	-0,45860	-0,49610	-0,49070
Indicators	**Wrrbm2**	**Wrrbm4**	**Wrrbms2**	**Wrrbms4**	**IBov**	**CDI**
Annualized Return	-0,03640	-0,00680	-0,08460	-0,03720	-0,11100	0,09390
Annualized Sd	0,16690	0,16430	0,17450	0,17870	0,19270	0,00460
Annualized Sharpe (Rf=9.24%)	-0,73270	-0,57840	-0,95460	-0,68860	-0,99100	0,00000
Worst Drawdown	0,15260	0,18010	0,19730	0,18390	0,24000	0,00000

Indicators (Cont.)	Wrrbm2	Wrrbm4	Wrrbms2	Wrrbms4	IBov	CDI
Calmar Ratio	-0,23850	-0,03760	-0,42860	-0,20220	-0,46300	∞

Search: Research results

We also checked the performance of the Ibovespa portfolio and the growth of the CDI as the free risk asset. Thus, the article could show a representation of a market portfolio and check how well it does and how efficient it is if an investor with lack of knowledge would come to use it. The indexed portfolio showed a low return with 16.67% and a negative Sharpe ratio, indicating a risky and inefficient method of investing.

5. Conclusions

This research aimed to verify the efficiency of the Volating Time (VT) and Reward to Risk Timing (RRT) strategies of portfolio selection proposed by Kirby and Ostidiek (2012a) applied on the Brazilian stock market, and then compare their results with the Naïve and Mean-Variance portfolios.

The results are inconclusive from the point of view of comparing strategies as a whole, but could indicate some very good conclusions. All of the strategies presented really low Sharpe Ratios which were mostly negative. Only 4 portfolios presented those positive indexes such as 6.6% for Wvm, 5.98% for VT4, 5.08% for RRT4 and 1.42% for RRTbm4. These also had the highest portfolios returns ranging from 63.8% for the RRTbm4 to 94.97% for the Wvm. However, the RRT portfolios presented high turnover, 18.09% for RRT4 and 11.28% for RRTbm4 while the VT4 features only 6.55%.

The results shown give the investor the idea that the best choice of portfolio will depend upon the economic setting that the Brazilian market is presented. During the first period RRTbm4 had the best results with 45.65% annualized return and a 223.7% Sharpe Ratio, the second period the RRTk4 went in the front with a 21.33% annualized return and a Sharpe Ration of 62.33%. The last period, everything changed again leaving no strategies with positive returns or Sharpe Ratios.

The focus of this research was to show better portfolios that preserve a low turnover, and the conclusion found was that only two portfolios followed these requirements were the VT4 and the Wvm portfolios. Besides the extensive information about the portfolio selection and their qualities which enabled comparison of their indicators, this research presents some limitations such as including only the assets that compose Ibovespa index and few strategies, being those great opportunities for future researches.

6. Referência

BM&FBOVESPA. Metodologia do Índice Bovespa: IBOVESPA. http://www.bmfbovespa.com.br/indices/ResumoIndice.aspx?Indice=Ibovespa&Idioma=pt-br . 11/4/2015.

CALDEIRA. J. MOURA. G.. SANTOS. A., Seleção de Carteiras Utilizando o Modelo Fama-French-Carhart. *RBE (Revista Brasileira de Economia).* Rio de Janeiro. v. 67. n. 1 . pp 45–65. Jan-Mar 2013.

CARHART. M. M., On Persistence in Mutual Fund Performance. *Journal of Finance*. v.52. p.57-82. 1997.

DEMIGUEL. V., GARLAPPI. L., AND UPPAL. R. Optimal versus naïve diversification: How inefficient is the 1/N portfolio strategy. *The Review of Financial Studies*, v 22, n 5, pp 1915–1953, 2009.

FAMA. E.F.; FRENCH. K. R. The Cross-section of Expected Stock Returns. *Journal of Finance*. v. 47. p.427-465. 1992.

FLETCHER. J.,. Do optimal diversification strategies outperform the 1/N strategy in U.K. stock returns?. *International Review of Financial Analysis* Vol. 20. pp 375-385. 2011.

IQUIAPAZA R.A., ROMA. C.M.S., CARVALHO. G.R., DRUMMOND. B.S.D., Desempenho de Novas Propostas de Seleção de Portfólios. *14º Encontro Brasileiro de Finanças*, jul de 2014.

KIRBY. C.. OSTDIEK. B.. It's All in the Timing: Simple Active Portfolio Strategies that Outperform Naïve Diversification. *Journal of Financial and Quantitative Analysis* Vol. 47. No. 2. pp. 437–467. 2012a.

KIRBY. C.. OSTDIEK. B.. Optimizing the Performance of Sample Mean-Variance Efficient Portfolios . *American Finance Association San Diego Meetings Paper*. Available at July 23. 2012b.

LEDOIT, O. WOLF, M. Improved Estimation of the Covariance Matrix of Stock Returns with an Application to Portfolio Selection**.** *Journal of Empirical Finance.* pp 603–621, out. 2003.

LEE. W., Risk-Based Asset Allocation: A New Answer to an Old Question?**.** *The Journal of Portfolio Management*. V.37. pp 11–28, Verão de 2011

LEVY, H., DUCHIN, R.. Markowitz's Mean-Variance Rule and the Talmudic Diversification Recommendation. In: Guerard, J. (Ed), *Handbook of portfolio Construction: Contemporary Aplications of Markowitz Techniques*. Anchorage USA: Springer Science & Business Media 12 de dez de 2009. pp 97-122

MADEIROS. M., PASSOS. A.. VASCONCELOS. G.. Parametric Portfolio Selection: Evaluating and Comparing to Markowitz Portfolios. *Revista Brasileirade Finanças (online)*. Rio de Janeiro. Vol. 12. No. 2. pp. 257–284. Jun 2014.

MAILLARD, S. RONCALLI, T. TEILETCHE, J., The Properties of Equally Weighted Risk Contribution Portfolios**.** *Journal of Portfolio Management*. V.36. pp 60-70, Verão de 2008

MARKOWITZ. H., Portfolio selection. *The Journal of Finance*. Vol. 7. No. 1.pp 77-91. 1952

SANTOS. A., TESSARI. C., Técnicas Quantitativas de Otimização de Carteiras Aplicadas ao Mercado de Ações Brasileiro. *Revista Brasileira de Finanças*. Vol. 10. No. 3. pp 369-393. 2012.

SHARMA. P.. Performance of risk based portfolios under different market conditions: Evidence from India. *Research in International Business and Finance* . mar 2015.

SHARPE. W. F., Capital asset prices: A Theory of Market Equilibrium Under Conditions of Risk. *Journal of Finance* Vol. 19. No 3. pp 425-442. 1964.

TU, J. ZHOU, C. Markowitz meets Talmud: A combination of sophisticated and naive diversification strategies. *Journal of Financial Economics.* Vol.99 . pp204-21. 2011.

---

i The dataset used in this study was updated within one year compared with Iquapaza's (2014).

[ii] See Fletcher (2011) for more information about the limited weight portfolios.