

**Área Temática: Finanças**

**EMPIRICAL EVIDENCE OF LONG-RANGE CORRELATIONS AND TESTING  
FOR NONLINEAR DEPENDENCE IN STOCK RETURNS**

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**Abstract**

Understanding stock markets price fluctuations is an important role in financial economics policy, corporate investment and financing strategies. The more efficient is the market, the more random is the sequence of price changes generated by the market. Therefore, the existence of long memory in stock market returns would affect the investment horizon of portfolio decisions. The principal purpose of this study is to research about long-range dependence in Latin American stock markets. Therefore, will be applied the BDS and K2K tests to verify the presence of some type of linear or nonlinear dependence, Hurst exponent and Detrended Fluctuation Analysis to verify the presence or not of the long-range dependence. This study provides a detailed examination of long-range dependence in weekly returns for the seven markets of Latin America from 1994 to 2005. The results provided from the BDS and K2K tests suggest the existence of nonlinear dependence in the returns series in all the analyzed countries. Also was evidenced random walk behavior for two markets and for the five other markets, the value the DFA test is smaller, which helps on the affirmative that the participation of long-range dependence in the formation of the prices weekly is minimum.

**Key words:** Latin American stock markets, Nonlinearity, Long-range dependence, Detrended fluctuation analysis.

## **1. Introduction**

The random walk hypothesis is one of the main influences in the growing literature in financial economics that analyses the temporal dependence of stock returns. The random walk hypothesis states that asset's returns are serially random thus it is independent of previous period's stock returns. The market that to present the more random sequence of prices is most efficient. Therefore, the existence of long memory in stock market returns would affect the investment horizon of portfolio decisions, which motivates the research on the subject.

In capital asset pricing, the process of scales, from days to decades, is a fundamental issue in financial economics. Pandey, Kohers and Kohers (1998) emphasize that random walk tests have been employed in the past to establish the weak form efficiency in the financial markets. However, methodologies and tests of other areas have gained prominence in such inquiries.

Researchers from heterogeneous areas of science have uncovered nonlinear deterministic long-range dependence that can generate data series that appear random to linear science. Lately, new statistical tests have been developed as, for example, Hurst (1951), Brock, Deschert and Scheinkman (1987), Peng et al. (1994) and Kocenda (2001). Because these tests can detect the presence of nonlinearity, short and long-term dependence, they are superior to the tests for linear autocorrelations used in the past.

The principal purpose of this study is to research about long-range dependence in Latin American stock markets. Therefore, will be applied the BDS and K2K tests to verify the presence of some type of linear or nonlinear dependence; Hurst exponent and Detrended Fluctuation Analysis to verify the presence or not of the long-range dependence. The database of the research is based on weekly stock prices indexes expressed in US dollar for the seven markets of Latin America obtained from the first week in January 1994 through October 2005.

The paper is organized as follows. First, will be presented a brief review of previous studies on long-range dependence in financial market. Next will be provided a description for the ARFIMA model, nonlinearities and long-range tests. In the following section the methodology and results are presented. The final part of the study discusses the results and concludes the paper.

## **2. Brief theoretical revision**

In the literature of financial economics exist different studies that investigate questions related to the dependence of short and long-term for financial time series. Crato and Lima (1994) had examined persistence in the conditional variance of US stock returns indexes. The results had shown evidence of long memory in high-frequency data, suggesting that models of conditional heteroskedasticity should be made of flexible enough to accommodate such dependences.

Pan, Liu and Bastin (1996) had examined the short and long-term behavior of foreign exchange rates, namely the British pound, the Canadian dollar, the Deutsche mark, the Japanese yen, and the Swiss franc. In this research, the authors had used the heteroskedasticity-robust variance-ratio test and the modified rescaled range analysis. The empirical results indicate the existence of short-term dependence for four of the five nominal,

and two of the five real exchange rate series examined, implying the rejection of the random walk hypothesis. Additionally, long-term persistence is found in the same four exchange rates that show short-term dependence, although the evidence for the British pound is marginal.

Grau-Carles (2000) examined the long-range dependence of the stock market returns using daily data of five indexes. The series examined include the Dow Jones, Standard & Poor 500, FTSE, Nykkei and the Indice General de la Bolsa de Madri – IGBM, where the applied methodology was Hurst exponent and DFA. The conclusion of the authors is no long-range dependence for returns, but strong dependence for absolute and squared returns. The results of the ARFIMA estimation for the  $d$  parameter are significant in all cases except for the squared returns of the FTSE. However, for the returns the value of  $d$  parameter is very small.

The degree of long-range dependence in indexes of the banking sector was examined in 41 different countries in Cajueiro and Tabak (2005). The results suggest that there is a stronger degree of long-range dependence in equity returns for emerging markets than for developed economies. Furthermore, on average long-range dependence in volatility seems to be stronger in developed economies than in emerging markets.

Generalized Hurst exponent was used for the examination of a wide variety of stock markets indexes in Matteo et al. (2005). They concluded that many indexes show sensitivity of the Hurst exponent to the degree of development of the market. United States, Japan, France and Australia, all the indexes presented  $H < 1/2$ . On the other hand,  $H > 1/2$  was calculated for Russian, Indonesian, Peru and other countries. They had suggested that this sensitivity of the scaling exponent comes to serve as a reference for a new and simple way of empirically characterizing the development of financial markets.

### **3. Long-range dependence and test**

The time dependence structure of stochastic process can be captured by the autocorrelation function. Characterized by autocorrelations at very high lags, long memory creates persistence (antipersistence) in the series over long time horizons. In these cases, autocorrelations declines at a slower hyperbolic rate, rather than at the exponential rate in the standard ARMA process. Mills (1999) suggested that many empirical observed time series, although appearing to satisfy the assumption of stationarity, with or without differentiation, seem to exhibit dependence between distant observations that, although small, is by no means negligible.

There exist many approaches of estimating long memory (fractional differencing). In financial time series, researchers have used the Autoregressive Fractionally Integrated Moving Average models ARFIMA (p,d,q). The ARFIMA models have the desired ability to match the slow decay of the autocorrelation functions. The difference of the ARFIMA (p,d,q) models to the ARIMA (p,d,q) is for not restricting the parameter  $d$  to an integer value, but rather allowing it to take an fractional value.

For non-integer values of  $d$ , the autocorrelations declines hyperbolically. When  $0 < d < 1/2$  the ARFIMA process is said to exhibit long memory with positive dependence. For  $d = 1$  the ARFIMA process is therefore identical to an ARIMA in that the autocorrelations decay exponentially. For  $d = 0$  the ARFIMA process reduces to an ARMA model and exhibit only short memory and, when  $-1/2 < d < 0$  the ARFIMA process exhibit long memory with negative dependence.

### 3.1 BDS and K2K tests for IID

The long-range dependence demands that some type of previous dependence in the data is identified. There has been a wide variety of nonlinear models for financial time series as, for example, ARCH and bilinear models. Mills (1999, 171), emphasizes that, with the growth of nonlinear models, it is not surprising that a large number of tests for nonlinearity have been proposed.

Brock, Deschert and Scheinkman (1987) have developed an approach that is more appropriated to the limited amount of data usually available in finance, the widely known as BDS test. The BDS test described in Hsieh (1989) is a portmanteau test based on the concept of the correlation integral developed by Grassberger and Procaccia (1983). For an observed series  $\{x_t\}$  of the size  $T$ , the correlation integral is defined  $C_{N,T}(\varepsilon) = 2 \sum_{t=1}^{T-N} I_t(x_t^N, x_{t+N}^N) / T_N(T_N - 1)$ . Where  $x_t^N = (x_t, x_{t+1}, \dots, x_{t+N-1})$  and  $x_{t+N}^N = (x_{t+N}, x_{t+N+1}, \dots, x_{t+2N-1})$  are called  $N$ -histories,  $I_t(x_t^N, x_{t+N}^N)$  is an indicator functional that if  $|x_t^N - x_{t+N}^N| < \varepsilon = 1$  and zero otherwise. That is, the correlation integral is a estimative of the probability that any two  $N$ -histories,  $x_t^N - x_{t+N}^N$ , are within  $\varepsilon$  of each other (close). If the data is IID then  $C_{N,T}(\varepsilon) \rightarrow C_{1,T}(\varepsilon)^N$ , as  $T \rightarrow \infty$  and,

$$BDS_{N,T} = \sqrt{T} (C_{N,T}(\varepsilon) - C_{1,T}(\varepsilon)^N) / s_{N,T}(\varepsilon) \quad [1].$$

Where  $s_{N,T}(\varepsilon)$  is an estimator of the asymptotic standard deviation of  $C_{N,T}(\varepsilon) - C_{1,T}(\varepsilon)^N$ . The BDS statistic is asymptotically standard normal under the IID null hypothesis. The rejection of this null hypothesis is consistent with some type of dependence in the data, which could result from a linear, nonlinear stochastic system or chaotic system.

Therefore, rejecting the IID hypothesis for  $\{x_t\}$  does not mean that  $\{x_t\}$  is predictable, but that  $\{(x^n)_t\}$  have some degree of predictability. The BDS statistics follows asymptotically a normal distribution with zero mean and unit variance under the null hypothesis of independence. In this manner, the BDS statistics converges in distribution to  $N(0,1)$  as  $T \rightarrow \infty$ , for  $\varepsilon > 0$  and  $N > 1$ , under the hypothesis of IID. Introduced for Kocenda (2001) the K2K test is a improved version of classic BDS test. The K2K test calculates the values of the correlation integral for 41 values of proximity parameter epsilon of standard deviation. This improves the precision in the evaluation.

### 3.2 Hurst exponent

An extensively approach for detecting the presence of long-range dependence is to use the range over standard deviation or rescaled range statistic (R/S). The pioneer in the study of long-range dependence in economic time series context was Mandelbrot (1971). However, R/S statistic was originally developed for Hurst (1951) in his studies of hydrological analysis. Campbell et al. (1997) shows that the R/S statistical is the range of partial sums of deviations of a times series from its mean, rescaled by its standard deviation. The rescaled range statistic is defined in [2].

$$(R/S)_T = \frac{1}{s_T} \left[ \text{Max}_{1 \leq k \leq T} \sum_{j=1}^k (r_j - \bar{r}_T) - \text{Min}_{1 \leq k \leq T} \sum_{j=1}^k (r_j - \bar{r}_T) \right] \quad [2].$$

For Equation [2], the value of  $s_T = [\sum (r_j - \bar{r}_T)^2]^{1/2}$  is the usual standard deviation;  $r_j$  is continuously compounded asset returns ( $r_1, \dots, r_T$ ) and  $\bar{r}_T$  denote the sample mean. Campbell et al. (1997) shows that the first term in [2] is the maximum of the partial sums of the first  $k$  deviations of  $r_j$  from the sample mean. Since the sum of the all  $T$  deviations of  $r_j$ 's from their mean is zero. The second term in [2] is the minimum of this same sequence of partial sums. The difference of the two quantities, called the range is always-nonnegative value and hence  $R/S \geq 0$ .

The value of Hurst exponent (H) is generally obtained from the linear regression over a sample of temporal growing  $\ln(R/S)_T = H \ln(T) + \ln(C)$ . An estimated value of  $H = 1/2$  means that the process has no long-range dependence, but if  $H > 1/2$  the series is persistent and is  $H < 1/2$  the series is considered antipersistent or mean reverting. Although it has long been established that the R/S statistic has the ability to detect long-range dependence. Lo (1991) showed that this statistic may be significantly biased when there is short-term dependence in the autocorrelation or heteroskedasticity and suggested the use of a modified rescaled range statistic. The difference between the Lo's modified statistic and the usual approach is the substitution of  $s_T$  for  $s_T(q)$  which becomes a consistent estimator for the variance of the partial sum in [2].

In this suggested method, the value of  $s_T(q)$  is given by  $s_T(q) = \sigma_T^2 + 2 \sum_{j=1}^q w_j(q) \hat{\gamma}_j$  where  $w_j(q) = 1 - [j/(q+1)]$  and  $\hat{\gamma}_j = 1/T [\sum_{i=j+1}^T (x_i - \bar{x}_T)(x_{i-j} - \bar{x}_T)]$  for all  $q > T$ . The weights  $w_j(q)$  are those suggested by Newey and West (1987). If  $q = 0$ , Lo's statistic reduce to Hurst's R/S statistic. In Teverloski et al. (1999) is showed that the Lo's statistic tends to reject the null hypothesis of no long-range dependence when the series is in fact long-range dependence and that the choice of the truncation lag  $q$  is crucial.

Thus, in order to detect short-range dependence in the data, the estimation of the Hurst exponent will be based on the adjusted time series for an Autoregressive processes with Exponential Generalized Autoregressive Conditional Heteroskedasticity proposed by Nelson (1991). This model is often given to acronym AR(p)-EGARCH(p,q) or for combinations of the Autoregressive and Moving Average models known as the ARMA(p,q), where  $p$  is lag operator the autoregressive and  $q$  is the lag operator of the moving average.

### 3.3 Detrended fluctuation analysis

In this study, we also seek to explore the method of detrended fluctuation analysis (DFA) which has proven useful in revealing the extent of long-range correlations in diverse time series. Regard for example Grau-Carles (2000), Király and Jánosi (2005). This method was been proposed by Peng et al. (1994) and improved by Viswanathan et al. (1997). The advantage of DFA over conventional methods (autocorrelations and Hurst exponent) is that it avoids spurious detection of apparent long-range correlation that is an artifact of non-stationary time series.

More recently, Király and Jánosi (2005) had demonstrated a form to calculate the DFA. In this manner, for a time series  $x_i (i = 1, \dots, N)$  with  $x_i$  as the increment of a random walk process

with  $\bar{x} = N^{-1} \sum_{i=1}^N x_i$  average it is calculated the profile  $y(j) = \sum_{i=1}^j [x_i - \bar{x}]$ . The profile  $y(j)$  is, divided thus into  $m$  non-overlapping segments of equal length  $s$ . In each segment, the local trend is fitted by a polynomial  $p_{s,m}^n$  of order- $p$  and the profile is detrended by subtracting this local fit  $y_{s,m}(i) = y(i) - p_{s,m}^n(i)$ ,  $j=1, \dots, N$ . For each segment  $m$  the squared fluctuation is calculated  $F_m^2(s) = 1/s \sum_{i=(m-1)s+1}^{ms} y_i^2$ . The squared fluctuation function of the process is estimated by averaging over all segments  $F^2(s) = 1/M \sum_{M=1}^M F_m^2(s)$ . For a more comprehensive and detailed description of the methodology is suggested the work of Kantelhardt et al. (2001) where the relationship between  $F(s)$  e  $n$  is given by

$$F(s) \sim n^\delta$$

[3].

Consequently, if  $\delta > 1/2$  long-range dependence positive is characterized (persistent), if  $\delta < 1/2$  the time series exhibit long-range dependence negative (antipersistent) and, if  $\delta = 0$ , the time series is random walk or uncorrelated.

#### 4. Methodology and preparation of the data

The database of the research is based in weekly stock prices indexes expressed in US dollar for the four largest markets of Latin America (Argentina-Merval, Brazil-Ibovespa, Chile-Igpa, Mexico-Ipc) and three smaller stock markets (Colombia-Igbc, Peru-Igbvl and Venezuela-Bboindex). All indexes are expressed in terms of the US dollar, facilitating cross-country comparison. The series of returns were collected from *Econômica* database in Brazil and have been carefully constructed to include dividend as well as capital gains.

The returns are calculated for making the logarithm differences between successive trading weeks ( $r_t = \log p_t - \log p_{t-1}$ ), where  $p_t$  denotes the closing prices of the stock index in week  $t$ . However, in order to avoid the Lo's critique and with the objective of eliminating possible influences of the dependence on short term, we use series of time filtered by a model AR(p)-EGARCH(p,q)-M and a model ARMA(p,q). In this investigation we have estimated all the possible models for  $p = 0, \dots, 8$  and  $q = 0, \dots, 3$ . The best model was selected using the Schwarz Information Criterion (SIC). After the filtering of the data, is expected that significant dependency in the behavior of the time series does not exist.

Descriptive statistics of the indexes are presented in Table 1. We use the weekly index level expressed in US dollar obtained from the first week in January 1994 through October 2005. The statistics include the average weekly lognormal returns, standard deviation, minimum and maximum values, kurtosis, skewness, autocorrelation, Jarque-Bera normality test and Phillips-Perron test.

Table 1. Descriptive statistics for the weekly stock markets returns (07 January 1994 to 28 October 2005)

Statistics	Argentina	Brazil	Chile	Colômbia	Mexico	Peru	Venezuela
Mean %	0.008	0.204	0.106	0.173	0.085	0.199	0.038
Maximum %	23.767	21.615	9.111	12.510	18.905	17.515	26.598
Minimum %	-57.724	-25.231	-11.397	-18.063	-38.042	-12.613	-31.186
Std. Dev.	5.685	6.308	2.422	3.581	4.605	3.127	5.305
Kurtosis	20.647	4.365	4.736	5.158	11.595	7.292	8.385

Skewness	-1.675	-0.494	-0.255	-0.053	-1.082	0.491	0.205
Jarque-Bera	8227.53	72.84	84.07	119.82	2016.37	497.56	737.61
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Autocorrelation (p)	0.090*	-0.091*	0.211**	0.176**	0.147**	0.136**	0.134**
Observations	612	616	616	616	616	616	607
Phillips-Perron	-23.05 <sup>++</sup>	-25.60 <sup>++</sup>	-20.40 <sup>++</sup>	-21.11 <sup>++</sup>	-21.56 <sup>++</sup>	-22.01 <sup>++</sup>	-23.05 <sup>++</sup>

\* Rejected null hypothesis of non autocorrelation at the 5% significance level;

\*\* Rejected null hypothesis of non autocorrelation at the 1% significance level;

<sup>++</sup> Rejected null hypothesis of a unit root at the 1% significance level.

The Latin America stock market returns series present significant kurtosis and skewness. In the case of kurtosis, all values are higher than the value of three for a normal distribution. High values of kurtosis imply that all empirical returns distributions series have fat tails compared with the normal distribution. A test for normality using Jarque-Bera statistics, which follows a chi-squared distribution with two degrees of freedom, rejected the normality for all market indexes in the sample period.

Finally, all significant Phillips-Perron's results indicate that the hypothesis of a unit root was rejected for each of the weekly returns series. This means that all stock indexes are integrated of order 1 ( $I-1$ ) or, equivalently, are non-stationary with the first difference stationary. These results corroborate with the conclusion obtained in Tsetsekos (1997) and Costa Jr. and Ceretta (2001).

## 5. Empirical results

The independence of the returns was rejected through the autocorrelation function, which indicates the existence of linear dependence in the data, therefore, is expected that the BDS and K2K test rejects the hypothesis of IID for the original data. Table 2 reports the BDS and K2K tests statistics. All stock markets returns are examined for embedding dimensions from 2 to 5 and a value of  $\varepsilon$  as  $\frac{1}{2}$  units of the standard deviation of the data.

Without exception, the BDS and the K2K tests yield to the rejection of linearity of the time series. The rejected null hypothesis of Independent and Identical Distribution for at least the five percent level to induce the possibility of predictable behavior in the researched data. The rejection of this null hypothesis of IID is consistent with some type of dependence in the time series; this could result from a linear stochastic system, a non-linear stochastic system or a non-linear deterministic system (chaotic). This analysis is valid for original returns, residuals of the AR(p)-EGARCH(p,q)-M and ARMA(p,q) models (filtered for short-range dependency).

Table 2. Values of BDS and K2K test for original and filtered returns series

Country	Embedding Dimension	Original Returns		Filtered returns			
		BDS test <sup>a</sup>	K2K Test	AR(p)-EGARCH(p,q)-M		ARMA(p,q)	
		BDS test <sup>a</sup>	K2K Test	BDS test <sup>a</sup>	K2K Test	BDS test <sup>a</sup>	K2K Test
Argentina	2	4.254**	1.376**	3.925**	1.379**	3.627**	1.388**
	3	5.640**	2.008**	5.012**	2.017**	5.181**	2.027**
	4	6.973**	2.607**	6.777**	2.617**	6.533**	2.627**
	5	7.684**	3.194**	7.432**	3.260**	7.384**	3.219**
Brazil	2	2.859*	1.441**	2.765*	1.442**	2.448*	1.458**

	3	5.058**	2.096**	4.988**	2.112**	4.667**	2.125**
	4	6.022**	2.726**	5.998**	2.715**	5.811**	2.761**
	5	7.909**	3.297**	7.996**	3.359**	7.987**	3.336**
<b>Chile</b>	2	6.727**	1.480**	5.867**	1.485**	4.302**	1.486**
	3	7.485**	2.171**	6.763**	2.203**	4.355**	2.205**
	4	7.359**	2.872**	5.231**	2.847**	3.596**	2.936**
	5	7.314**	3.560**	5.141**	3.599**	4.154**	3.645**
<b>Colombia</b>	2	5.963**	1.270**	5.723**	1.269**	5.064**	1.265**
	3	7.234**	1.821**	6.912**	1.834**	5.699**	1.837**
	4	8.004**	2.349**	7.740**	2.355**	6.386**	2.392**
	5	8.512**	2.873**	7.313**	2.922**	6.857**	2.945**
<b>Mexico</b>	2	5.954**	1.354**	6.366**	1.268**	6.259**	1.371**
	3	7.577**	1.964**	7.619**	1.987**	7.755**	1.587**
	4	8.857**	2.546**	8.996**	2.558**	9.121**	2.579**
	5	10.189**	3.105**	9.852**	3.139**	10.252**	3.149**
<b>Peru</b>	2	5.760**	1.336**	4.657**	1.347**	3.166**	1.328**
	3	7.621**	1.932**	6.823**	1.933**	4.658**	1.943**
	4	8.064**	2.522**	6.848**	2.536**	5.212**	2.548**
	5	8.698**	3.099**	6.778**	3.009**	6.141**	3.142**
<b>Venezuela</b>	2	3.545**	1.209**	3.428**	1.211**	3.084**	1.239**
	3	5.770**	1.720**	5.554**	1.725**	5.400**	1.762**
	4	7.354**	2.186**	7.293**	2.198**	6.966**	2.234**
	5	8.780**	2.615**	8.591**	2.635**	8.392**	2.667**

a.  $\mathcal{E} = 1/2$ ;

\* Rejected null hypothesis of Independent and Identical Distribution the 5% significance level;

\*\* Rejected null hypothesis of Independent and Identical Distribution at the 1% significance level.

The fact that the BDS and the K2K test rejected the IID hypothesis for the filtered data of linear effects of short term causes a concern about the performance of AR(p)-EGARCH(p,q)-M and ARMA(p,q) models in supplying a good adjustment to the data. In this manner, it was of if waiting that it did not have more traces of previsibility in the data. However, the rejection of IID for filtered returns of linear effects is consistent with the more general version of the random walk hypothesis. This version is known as random walk 3 (RW3). Campbell et al. (1997-pp.32-33) discover that RW3 is any process for which  $Cov[\varepsilon_t, \varepsilon_{t-k}] = 0$  for all  $k \neq 0$ , but being possible  $Cov[\varepsilon_t^2, \varepsilon_{t-k}^2] \neq 0$  for some  $k \neq 0$ .

It's clear that the rejection of the hypothesis that stock returns are IID doesn't contradict market efficiency. Market efficiency implies that forecast errors of returns are not predictable. The fact that the square of the residues is strong correlated and this is one of the reasons to take BDS and K2K test to reject IID hypothesis and says nothing about the predictable of forecast errors. Thus, BDS and K2K rejection doesn't necessary implies that a stock market returns series has a time varying conditional mean. But it could be evidence for a time varying conditional variance (ARCH, GARCH or FIGARCH). Therefore, in order to verify the existence or not of long-range dependence on the stock returns, the execution of specific tests as Hurst and DFA are needed.

Table 3 reports the results for the application of the Hurst exponent for the researched stock market returns. In the left side of the table, the first columns shows the Hurst exponent for original returns and second column presents Wald test for the null hypothesis  $H = 1/2$ . The right side shows these values for filtered returns of the AR(p)-EGARCH(p,q)-M and ARMA



models. Although, more important than to test this hypothesis, is to verify the magnitude of exponent.

In Table 3 is showed the average Hurst exponent which is 0.634 (original returns), 0.597 (filtered returns for AR(p)-EGARCH(p,q)-M and 0.586 (filtered returns for ARMA(p,q). With the exception of Venezuela, for filtered returns, the Hurst exponents are bigger than  $\frac{1}{2}$ , which is characterized as long-range dependence, long memory or with positive correlation (persistent).

For the filtered returns of the Venezuela, we estimated a exponent non different from  $\frac{1}{2}$  which provides an indication of a random walk series, free of the long-range dependence. The values these results are smaller than the ones obtained in Cajueiro and Tabak (2005) for daily returns series in Argentina, Brazil, Chile and Mexico.

Table 3. Values of Hurst exponent for original and filtered returns series

Country	Original returns		Filtered returns					
	Hurst	Wald Test	model	Hurst	Wald test	Model	Hurst	Wald test
<b>Argentina</b>	0.617**	37.920	AR(1)	0.608**	30.157	ARMA(1,1)	0.587**	19.464
<b>Brazil</b>	0.607**	18.318	AR(8)	0.629**	42.427	ARMA(2,2)	0.606**	26.736
<b>Chile</b>	0.693**	93.123	AR(3)	0.589**	24.721	ARMA(1,1)	0.610**	29.950
<b>Colombia</b>	0.672**	28.891	AR(2)	0.629**	16.379	ARMA(1,1)	0.593**	7.309
<b>Mexico</b>	0.583**	13.023	AR(1)	0.572**	20.850	ARMA(1,0)	0.565**	18.530
<b>Peru</b>	0.655**	41.710	AR(3)	0.578**	22.323	ARMA(1,1)	0.573**	24.329
<b>Venezuela</b>	0.610*	5.718	AR(2)	0.571	2.863	ARMA(2,0)	0.566	2.481
<b>Mean</b>	0.634			0.597			0.586	

\* Rejected null hypothesis of  $H = \frac{1}{2}$  at the 5% significance level;

\*\* Rejected null hypothesis of  $H = \frac{1}{2}$  at the 1% significance level.

Table 4 presents the results of the DFA for various stock markets return series researched. This technique always yields smaller values than the ones obtained with the method traditional Hurst exponent analysis for the original and filtered returns. It's possible to see a wide variety of behaviors. It has been found evidence that the values of the test are affected by short-term dependence; therefore, the results for filtered returns are always smaller. It seems that the stock markets filtered returns series for a ARMA(p,q) can be put into three different categories.

Table 4. Values of DFA for original and filtered returns series

Country	Original returns		Filtered returns					
	DFA	Wald Test	Model	DFA	Wald test	Model	DFA	Wald test
<b>Argentina</b>	0.545**	56.250	AR(1)	0.523**	18.070	ARMA(1,1)	0.489*	5.252
<b>Brazil</b>	0.523**	33.063	AR(8)	0.543**	198.760	ARMA(2,2)	0.530**	59.172
<b>Chile</b>	0.611**	251.449	AR(3)	0.464**	75.981	ARMA(1,1)	0.482**	22.438
<b>Colombia</b>	0.552**	16.000	AR(2)	0.470**	14.121	ARMA(1,1)	0.436**	45.385
<b>Mexico</b>	0.550**	17.361	AR(1)	0.515	2.891	ARMA(1,0)	0.507	0.579
<b>Peru</b>	0.573**	44.041	AR(3)	0.469**	22.505	ARMA(1,1)	0.465**	15.465
<b>Venezuela</b>	0.579**	51.579	AR(2)	0.516	1.659	ARMA(2,0)	0.509	0.465
<b>Mean</b>	0.562			0.500			0.488	

\* Rejected null hypothesis of  $DFA = 1/2$  at the 5% significance level;

\*\* Rejected null hypothesis of  $DFA = 1/2$  at the 1% significance level.

The first category is having a DFA larger than  $1/2$ , which indicates long-range dependence with persistent behavior. In this classification is located the Brazilian stock market. A second category represents the stock markets with DFA non-different than  $1/2$ . This is the case of Mexico and Venezuela, which indicates uncorrelated white noise behavior. Finally, the third category represents the stock markets with a DFA smaller than  $1/2$ , which indicates long-range dependence with negative correlation (antipersistent behavior). In this classification, is located the stock markets of Argentina, Chile, Colombia and Peru.

## 6. Conclusion

This study provides a comprehensive empirical analysis of the Latin America stock markets connected with long-range dependence. We have studied weekly stock markets returns of the seven countries from January 1994 until October 2005. The returns series were obtained from *Economática* database in Brazil. This study was carried out using original returns and filtered returns series. The filtering of the data was obtained through the adjustment of autoregressive and autoregressive moving average models.

The results provided from the application of BDS and K2K tests suggest the existence of nonlinear dependence in the returns series in all the analyzed countries. A possible explanation for these results is the dynamics of Autoregressive Conditional Heteroskedastic models applied on the square of the errors. For this form, these results don't affect the market efficiency but they corroborate for the inquiry of long-range dependence.

The individual analysis of the Hurst exponent identifies strong evidence of long-term dependence for all the countries, excluded Venezuela. Therefore, Latin America stock markets present similar and persistent behaviors. These results must be analyzed with attention, since the Hurst exponent is sensible to Lo's critique. Moreover, another precaution to be made in the analysis of the conclusion of this paper is that the filtering of the series may not have been efficient, leaving traces of short-term predictability on the researched data.

With the application of DFA methodology is evident three distinct behaviors in the Latin America stock markets. First, the persistent behavior suggesting long-range dependence with positive correlation (Brazil). Second, white noise behavior consistent with the market efficiency (Mexico and Venezuela). Third, antipersistent behavior suggesting long-range dependence with negative correlation (Argentina, Colombia Chile and Peru).

These results suggest that the behavior of the returns of the Latin America stock markets is consistent with the market efficiency. Therefore, for two markets was evidenced random walk behavior and for the five other markets, the value of the DFA is smaller, which helps on the affirmative that the participation of long-range dependence in the formation of the prices weekly is minimum. Additionally, in this market, a simple process of diversification becomes perfectly consistent with the market efficiency, fact identified for the average DFA.

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